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Derivation of finite-size scaling for mean-field models from the Burgers equation

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Abstract. The magnetization per spin for a class of models with the Husimi–Temperley type interaction satisfies the Burgers equation with a diffusion coefficient $1/2N$, where N is the number of particles. The role of time is played by the dimensionless interaction parameter, and the role of spatial coordinate by the dimensionless field variable. Both thermodynamic scaling and finite-size scaling for the magnetization near the critical point are derived from a family of self-similar solutions of the corresponding Burgers equation. The models are specified by the initial conditions which are chosen to correspond in the high-temperature region to a vanishing interaction constant and in the low-temperature region to an infinitely large interaction constant.

1. Introduction

In the rigorous theory of phase transitions and critical phenomena there is an approach based on partial differential inequalities involving the order parameter, see, e.g., [1-3]. This approach originates from the well known Griffiths–Hurst–Sherman inequalities [4] and their generalizations [5]. It makes possible the rigorous analysis of the global phase structure of specific classes of model systems, the derivation of inequalities for the critical exponents etc, even in the absence of exact solutions. As pointed out by Aizenman [1], the efficiency of studying systems with an infinite number of degrees of freedom by using partial differential inequalities with respect to few relevant variables may be attributed to the existence of the Kadanoff–Wilson [6, 7] renormalization group transformation.

In our paper [8] it has been shown that an exact partial differential equation for the order parameter of a finite system may be derived in the simplest case of the Husimi–Temperley–Ising model. Here we generalize that treatment to the whole class of models described by a Hamiltonian of the form

$$\mathscr{H}_N(\{\sigma_i\}) = -\frac{J}{2N}\sum_{i,j=1}^{N}\sigma_i\sigma_j - H\sum_{i=1}^{N}\sigma_i \qquad (1)$$

where $\{\sigma_i \in \mathbb{R}^1, i = 1, \ldots, N\}$ are the dynamical variables, $J \geqslant 0$ is the interaction constant, $H \in \mathbb{R}^1$ is an external magnetic field. Ellis and Newman [9] have considered the

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class of models with a joint probability distribution of the spins $\{\sigma_i\}$ given by the measure μ_ρ on $\mathbb{R}^N$ of the form

$$\mu_\rho(\mathrm{d}x_1, \dots, \mathrm{d}x_N|K, h) = [Z_N^{(\rho)}(K, h)]^{-1} \exp\left[\frac{K}{2N}\left(\sum_{i=1}^{N} x_i\right)^2 + h \sum_{i=1}^{N} x_i\right] \mu_0(\mathrm{d}x_1, \dots, \mathrm{d}x_N|\rho). \tag{2}$$

Here $K = J/k_B T$ and $h = H/k_B T$ are dimensionless thermodynamic parameters, μ_0 is a free measure of the product type

$$\mu_0(\mathrm{d}x_1, \dots, \mathrm{d}x_N|\rho) = \prod_{i=1}^{N} \rho(\mathrm{d}x_i) \tag{3}$$

where ρ is an arbitrary Borel probability measure on $\mathbb{R}^1$ which satisfies the condition

$$\int_{\mathbb{R}^1} \mathrm{e}^{x^2} \rho(\mathrm{d}x) < \infty. \tag{4}$$

The normalization coefficient $Z_N^{(\rho)}(K, h)$ in (2) is the partition function of the model (1).

Here we do not confine ourselves to free product measures (3), (4), but extend the consideration by including the spherical model [10], which has a free measure of the form

$$\mu_0^{\mathrm{S}}(\mathrm{d}x_1, \dots, \mathrm{d}x_N) \sim \delta\left(\sum_{i=1}^{N} x_i^2 - N\right) \prod_{i=1}^{N} \mathrm{d}x_i \tag{5}$$

with $\mathrm{d}x$ being the Lebesgue measure on $\mathbb{R}^1$, as well as its generalizations in the spirit of [11, 12].

Let us formally introduce a 'time' variable t and a 'spatial' coordinate x by the equalities

$$t = K - K_c \qquad x = -h \tag{6}$$

where $K_c > 0$ is a parameter to be determined below. Consider the magnetization per spin of a finite system of N particles

$$m_N^{(\rho)}(t, x) := \int_{\mathbb{R}^1} \left(N^{-1} \sum_{i=1}^{N} x_i\right) \mu_\rho(\mathrm{d}x_1, \dots, \mathrm{d}x_N|K_c + t, -x). \tag{7}$$

By differentiating (7) with respect to the variables t and x, and taking into consideration the explicit expression (2) for the measure μ_ρ, we obtain that $m_N^{(\rho)}(t, x)$ obeys the well known Burgers equation (see, e.g., [13])

$$\frac{\partial}{\partial t} m + m \frac{\partial}{\partial x} m = \frac{1}{2N} \frac{\partial^2}{\partial x^2} m. \tag{8}$$

Note that the diffusion coefficient $1/2N$ in (8) is inversely proportional to the number of particles N in the system. Various models of the Husimi–Temperley class under consideration differ only in the initial condition:

$$m_N^{(\rho)}(t = -K_c, x) \equiv \Phi_N^{(\rho)}(x) = [Z_N^{(\rho)}(0, -x)]^{-1} \int_{\mathbb{R}^N} \left(N^{-1} \sum_{i=1}^{N} x_i\right) \exp\left(-x \sum_{i=1}^{N} x_i\right) \mu_0(\mathrm{d}x_1, \dots, \mathrm{d}x_N|\rho) \tag{9}$$

which depends on the choice of the free measure μ_0.

On the basis of the Burgers equation (8) with the initial condition $\Phi_N^{(\rho)}(x) = -\tanh(x)$ at time $t = -1$, which corresponds to the Husimi–Temperley–Ising model, in our paper

[8] an analogy has been drawn between the appearance in the thermodynamic limit of a first-order phase transition with respect to the field variable x at $K \geqslant K_c = 1$ and the development of a shock wave at time $t = 0$ when the limit $N \to \infty$ on the right-hand side of equation (8) is taken. Indeed, the steepness of the front of the solution $m = m_N(t, x)$ of equation (8) with the given initial condition $-\tanh(x)$ reaches a maximum at the point $x = 0$, where it is proportional to the initial magnetic susceptibility of the system. The evolution of the initial condition in the time interval $-1 < t < 0$ is such that the steepness of the front remains finite even in the thermodynamic limit $N \to \infty$. At time $t = 0$, however, the steepness of the front at $x = 0$ (the initial magnetic susceptibility) increases unboundedly when $N \to \infty$, i.e. when the diffusion coefficient in equation (8) tends to zero. Next, at times $t > 0$ and in the limit $N \to \infty$, the nonlinear growth of the steepness leads to the formation of a shock wave which describes the jump of the magnetization per spin under the change of the magnetic field variable x across the point $x = 0$.

In our paper [8] the questions about the existence of self-similar solutions of the Burgers equation and their relevance to finite-size scaling laws near a phase transition point have not been studied. This is the main purpose of the present work. In section 2 we obtain thermodynamic scaling for the magnetization from the self-similar solutions of the Cauchy problem (8), (9) in the limit $N \to \infty$. In section 3 we derive a one-parameter family of critical finite-size scaling laws compatible with the Burgers equation. It is shown that the mean-field finite-size scaling law is uniquely determined by the thermodynamic initial condition. A short discussion of the results is given in section 4.

2. Self-similar solutions and thermodynamic scaling laws

The thermodynamic scaling (see, e.g., [14]) predicts that in the neighbourhood of the critical point $t = 0$, $h = 0$, see (6), the bulk magnetization per spin $m_\infty(t, x)$ is a generalized homogeneous function of the variables t and x of the form

$$m_\infty(t, x) \simeq |t|^\beta v_\pm(x|t|^{-\Delta}) \tag{10}$$

where the two branches $v_\pm(\ )$ of the scaling function, corresponding to $t \lessgtr 0$, may be different. Here by β and $\Delta = \beta + \gamma$ we denote the standard critical exponents [14]. Note that the variable t, see (6), differs in sign from the commonly accepted definition, therefore the high-temperature region $T_c < T \leqslant \infty$ corresponds to negative t, $-K_c \leqslant t < 0$, and the low-temperature region $0 \leqslant T < T_c$ corresponds to positive t, $0 < t \leqslant \infty$.

Since $m_\infty(t, x)$ obeys equation (8) in the limit $N \to \infty$:

$$\frac{\partial}{\partial t} m + m \frac{\partial}{\partial x} m = 0 \tag{11}$$

then, if scaling holds, the expression (10) should be among the self-similar solutions of equation (11), at least locally in the neighbourhood of the critical point. Let us consider that question in more detail.

2.1. *The high-temperature region* $-K_c \leqslant t < 0$

In this case we look for self-similar solutions of equation (11) of the form

$$m = (-t)^\beta v_+(\xi) \qquad \xi = x(-t)^{-\Delta} \tag{12}$$

with arbitrary positive exponents β and Δ. By inserting (12) into (11) we obtain an ordinary differential equation for the unknown function $v_+(\xi)$:

$$[(-t)^{\beta+1-\Delta}v_+ + \Delta\xi]v'_+ - \beta v_+ = 0. \tag{13}$$

Obviously, this equation defines a function of ξ only if the following self-similarity condition holds:

$$\Delta = \beta + 1. \tag{14}$$

Since by definition $\Delta = \beta + \gamma$, it follows that self-similar solutions of (11) exist only for the mean-field value $\gamma = 1$ of the susceptibility critical exponent. Then, integrating equation (13) we obtain that $v_+ = v_+(\xi)$ is defined as an implicit function of ξ by the equation

$$|v_+| = A|v_+ + \xi|^{\beta/(\beta+1)}. \tag{15}$$

Here $A > 0$ is an arbitrary integration constant and the exponent β may still take any positive value.

Let us now take into consideration that the magnetization (12) must obey the initial condition (9) in the limit $N \to \infty$. Hence we obtain the following condition on the function v_+:

$$(K_c)^\beta v_+(xK_c^{-\beta-1}) = \Phi_\infty^{(\rho)}(x). \tag{16}$$

We may point out, for example, that in the case of the Husimi–Temperley–Ising model one has

$$\Phi_N^{(1)}(x) = -\tanh(x) \tag{17}$$

and in the case of the Husimi–Temperley spherical model the initial condition is

$$\begin{aligned}\Phi_N^{(S)}(x) &= \int_{\mathbb{R}^N} \left(N^{-1}\sum_{i=1}^N x_i\right)\delta\left(\sum_{i=1}^N x_i^2 - N\right)\exp\left(-x\sum_{i=1}^N x_i\right)\mathrm{d}x_1\ldots\mathrm{d}x_N \\ &= -I_{N/2}(Nx)/I_{(N-2)/2}(Nx) = -2x[1+(4x^2+1)^{1/2}]^{-1} + \mathcal{O}(N^{-1}).\end{aligned} \tag{18}$$

Obviously, neither $\Phi_N^{(1)}(x)$, nor $\Phi_N^{(S)}(x)$ obeys in general the constraint imposed on $\Phi_N^{(\rho)}(x)$ by equation (15) at $t = -K_c$:

$$|\Phi_\infty^{(\rho)}(x)| = A|K_c\Phi_\infty^{(\rho)}(x) + x|^{\beta/(\beta+1)}. \tag{19}$$

Moreover, from (19) it follows that the function $\Phi_\infty^{(\rho)}(x)$ must diverge when $x \to \infty$, while actually $\Phi_\infty^{(\rho)}(x \to \pm\infty) \to \mp m_0(\infty)$, where $m_0(\infty)$ is the saturation magnetization per spin. Therefore, the models of the Husimi–Temperley class considered here may not have globally self-similar magnetization of the form (12). Consider then the local properties of the solutions of the Cauchy problem when $x \to 0$. Confining ourselves to the case of symmetric free measures μ_0, which implies $\Phi_N^{(\rho)}(-x) = -\Phi_N^{(\rho)}(x)$ and assuming the analyticity of $\Phi_\infty^{(\rho)}(x)$ at the point $x = 0$, we consider the expansion

$$\Phi_\infty^{(\rho)}(x) = \frac{\partial\Phi_\infty^{(\rho)}(0)}{\partial x}x + \frac{1}{3!}\frac{\partial^3\Phi_\infty^{(\rho)}(0)}{\partial x^3}x^3 + \ldots \qquad (x \to 0). \tag{20}$$

Now we may satisfy the constraint (19) by setting

$$\beta = \tfrac{1}{2} \qquad K_c^{-1} = -\frac{\partial\Phi_\infty^{(\rho)}(0)}{\partial x} > 0 \qquad A^{-3} = K_c^4\left|\frac{1}{3!}\frac{\partial^3\Phi_\infty^{(\rho)}(0)}{\partial x^3}\right|. \tag{21}$$

For the coefficients of higher powers of x one obtains a system of recursion relations.

Thus the condition that the self-similar solution (12) locally satisfies the initial condition (20) in the neighbourhood of the point $x=0$ implies the mean-field value $\beta=\frac{1}{2}$ of the magnetization critical exponent. Moreover, we have expressed the parameters K_c and A in terms of quantities depending on the specific model.

2.2. *The low-temperature region* $0<t\leqslant\infty$

In this case we look for self-similar solutions of equation (18) of the form

$$m=t^{\beta}v_{-}(\xi) \qquad \xi=xt^{-\Delta} \tag{22}$$

with arbitrary positive exponents β and Δ. By inserting (22) into (11) we obtain an ordinary differential equation for the unknown function $v_{-}(\xi)$ (cf (13))

$$[t^{\beta+1-\Delta}v_{-}-\Delta\xi]v_{-}'+\beta v_{-}=0. \tag{23}$$

The self-similarity condition is again given by (14), hence $\gamma=1$. The function $v_{-}=v_{-}(\xi)$ is defined as an implicit function of ξ by the equation (cf (15))

$$|v_{-}|=A|v_{-}-\xi|^{\beta/(\beta+1)} \tag{24}$$

where $A>0$ is an arbitrary integration constant.

Since $t=0$ is a point of singularity, a new Cauchy problem now emerges with the natural initial (under time inversion) condition being set at $t=\infty$, which corresponds to an infinitely large interaction constant or zero temperature. In this limit the magnetization per spin should tend to the step function

$$\lim_{t\to\infty} m_{\infty}(t,x)=-\mathrm{sgn}(x)m_0(\infty) \tag{25}$$

where $m_0(\infty)$ is the saturation magnetization per spin. From (22) and (25) it follows that when $\xi\to 0^{\pm}$ the function $v_{-}(\xi)$ should have a singular behaviour of the form

$$v_{-}(\xi)\simeq -\mathrm{sgn}(\xi)m_0(\infty)|\xi|^{\beta/(\beta+1)}. \tag{26}$$

However, functions with such an asymptotic behaviour cannot satisfy equation (24) at small ξ, i.e. we find again that a globally self-similar solution of the Cauchy problem (11), (25) does not exist. Obviously, the restriction of our consideration to the neighbourhood of the line $\{0<t\leqslant\infty, h=0\}$ does not suffice now. To satisfy the self-similarity constraints, we have to set the initial condition closer to the critical point $t=0^{+}$, $h=0$. This, in turn, necessitates the use of a phenomenological argument about its shape. By taking into account the existence of spontaneous magnetization $m_{\infty}(t,0^{\pm})=\mp m_0(t)$ and initial magnetic susceptibility $\chi_{\infty}(t,0^{\pm})=\chi_0(t)$ when $t>0$ and $H\to 0^{\pm}$, we assume that for sufficiently small $t_0>0$ and $H\to 0^{\pm}$ the initial condition has the form

$$m_{\infty}(t_0,H/k_BT_0)\simeq \mathrm{sgn}(H)m_0(t_0)+\chi_0(t_0)H+\mathcal{O}(H^3). \tag{27}$$

Note that when $t_0\to 0$

$$m_0(t_0)\sim t_0^{\beta}\to 0 \qquad \chi_0(t_0)\sim t_0^{-\gamma}\to 0 \qquad (t_0\to 0) \tag{28}$$

in accordance with the definition of the critical exponents $\beta>0$ and $\gamma>0$. Next, (24) may be rewritten as an equation of state for the magnetization (22):

$$|tm-x|=|m/A|^{(\beta+1)/\beta}. \tag{29}$$

By inserting the initial condition (27) into equation (29) at $t=t_0$ and $x\to 0$ and by comparing terms of the same order of magnitude in the field H, we obtain

$$m_0(t_0)\simeq A^{\beta+1}t_0^{\beta} \qquad (k_BT_0)\chi_0(t_0)\simeq \beta t_0^{-1}. \tag{30}$$

Therefore, the self-similar solution (22) satisfies the local initial condition (27) at arbitrary positive values $\beta > 0$ of the spontaneous magnetization critical exponent and the mean-field value $\gamma = 1$ of the initial susceptibility critical exponent. But to comply with the thermodynamics of the Husimi–Temperley models we have to set

$$\beta = \tfrac{1}{2} \qquad A = \left(\lim_{t \to 0} t m_0^2(t) \right)^{1/3}. \tag{31}$$

Then expressions (30) for the spontaneous magnetization and the initial susceptibility take the asymptotic form predicted by the mean-field theory.

3. Derivation of finite-size scaling from the Burgers equation

Consider now what kind of information about critical finite-size scaling is contained in the form of the differential equation for the order parameter of the finite system, see (8), and what information is contained in the initial condition (9).

In accordance with the Privman hypothesis [15], see also [16], we look for solutions of the Burgers equation (8) in the self-similar form

$$m = N^{-p} w(N^q t, N^r x) \tag{32}$$

with arbitrary positive exponents p, q and r. By introducing the variables

$$w = N^p m \qquad x_1 = N^q t \qquad x_2 = N^r x \tag{33}$$

we may cast the Burgers equation (8) in the form

$$\frac{\partial}{\partial x_1} w + N^{-p-q+r} w \frac{\partial}{\partial x_2} w = \tfrac{1}{2} N^{-1-q+2r} \frac{\partial^2}{\partial x_2^2} w. \tag{34}$$

Since the function $w(x_1, x_2)$, see (32), does not explicitly depend on N, the existence of such a self-similar solution requires the equalities

$$q = 1 - 2p \qquad r = 1 - p \tag{35}$$

where $0 < p < \frac{1}{2}$ is an arbitrary parameter at present. Therefore, the most general form of the finite-size scaling laws compatible with the Burgers equation is

$$m_N(t, x) = N^{-p} w(N^{1-2p} t, N^{1-p} x) \qquad 0 < p < \tfrac{1}{2}. \tag{36}$$

Here the function $w(x_1, x_2)$ is a solution of the N-independent Burgers equation

$$\frac{\partial}{\partial x_1} w + w \frac{\partial}{\partial x_2} w = \frac{1}{2} \frac{\partial^2}{\partial x_2^2} w. \tag{37}$$

Let us now consider the initial condition (9) which in the new variables (33) has the form

$$w(x_1 = -K_c N^{1-2p}, x_2) = N^p \Phi_N^{(p)}(N^{p-1} x_2). \tag{38}$$

To study (38) when $N \to \infty$, we need the asymptotic form of $w(x_1, x_2)$, when $x_1 \to -\infty$. We note that at any fixed $t \neq 0$ the limit $N \to \infty$ in expression (36) for the magnetization should lead to the thermodynamic result (10) which may happen only if

$$w(x_1, x_2) \underset{x_1 \to \pm\infty}{\simeq} |x_1|^{\beta} v_{\mp}(x_2 |x_1|^{-\beta-1}) \tag{39}$$

and, moreover, under the condition that p and β satisfy the equalities

$$1-p=(1-2p)(\beta+1) \qquad (1-2p)\beta=p. \tag{40}$$

Hence we obtain

$$p=\beta/(2\beta+1) \tag{41}$$

or in view of $\beta=\frac{1}{2}$ we have $p=\frac{1}{4}$. Now one may readily check that the initial condition (38) at fixed x_2 and $N\to\infty$ reduces to the equality

$$(K_c)^\beta v_+(0)=\Phi_\infty^{(\rho)}(0) \tag{42}$$

which holds due to condition (16) at $x=0$.

4. Discussion

We have derived the general one-parameter family (36) of finite-size scaling laws compatible with the Burgers equation. Remarkably, the value $\beta=\frac{1}{2}$ of the magnetization critical exponent, and hence the value $p=\frac{1}{4}$, see (41), is determined solely by the power of the second term in the Taylor expansion (20) of the initial condition and does not depend on details of the model, i.e. on the specific choice of the free measure μ_0. If instead of (20), the following expansion holds:

$$\Phi_\infty^{(\rho)}(x)=\frac{\partial\Phi_\infty^{(\rho)}(0)}{\partial x}x+\frac{1}{(2k+1)!}\frac{\partial^{2k+1}\Phi_\infty^{(\rho)}(0)}{\partial x^{2k+1}}x^{2k+1}+\ldots \tag{43}$$

we will obtain

$$\beta=(2k)^{-1} \qquad p=[2k(k+1)]^{-1}. \tag{44}$$

In the thermodynamic limit equation (11) describes the exact evolution of the magnetization profile $\{m_\infty(t,x), x\in\mathbb{R}^1\}$ under the change of the interaction parameter $K=K_c+t$. Two Cauchy problems have emerged in a natural way. In the high-temperature region $0\leqslant K<K_c$ the initial moment of time $t=-K_c$ corresponds to a system of non-interacting spins, the magnetization profile of which carries information about the limiting free measure μ_0 on $\mathbb{R}^\infty$. In the case when μ_0 is a product measure [9], see (3), the thermodynamic limit for the system of non-interacting particles is trivial, the initial condition (9) is independent of N and characterizes only the one-particle measure. Then equation (11) relates the self-similar behaviour of the system of interacting particles in the neighbourhood of the critical point to the trivial behaviour of the system of free particles. In that sense the evolution equation (11) corresponds to the renormalization group flow from the neighbourhood of the critical point to the trivial fixed point characterized by vanishing interaction constant. In an analogous way, in the low-temperature region $K_c<K\leqslant\infty$ the evolution equation (11) corresponds to the renormalization group flow from the neighbourhood of the critical point to another trivial fixed point characterized by the infinitely large interaction constant.

In both cases self-similarity appears as a local property of the Cauchy problem. The role of the initial condition consists in the reduction of the one-parameter family of self-similar solutions of the evolution equation to a single representative. The classes of critical universality are determined by the gross features of the initial condition near the zero-field point, and in the low-temperature region they are also determined by the asymptotic behaviour of the order parameter on approaching the critical temperature.

For any finite system of the Husimi–Temperley class the one-parameter family of solutions (36) of the Burgers equation analytically depends on the temperature, $-K_c \leq t < \infty$, and the magnetic field $x \in \mathbb{R}^1$ with the only exception of the point at infinity $t = \infty$, $x = 0$. The reduction of the parameter $p \in (0, \frac{1}{2})$ of the corresponding family of finite-size scaling laws to its value $p = \beta/(2\beta + 1)$ takes place under the condition that the thermodynamic limit for the magnetization exists. The explicit expression for the finite-size scaling function $w(x_1, x_2)$ may be easily obtained from the well known integral representation for the solution of the Cauchy problem for the Burgers equation, see, e.g., [13].

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